

DO NOW

Find $\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ given $f(x) = x^2 - 2x$

$$\begin{aligned} \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^2 - 2(x+\Delta x) - (x^2 - 2x)}{\Delta x} \\ \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 2x - 2\Delta x - x^2 + 2x}{\Delta x} \\ \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x - 2\Delta x + (\Delta x)^2}{\Delta x} \\ \lim_{\Delta x \rightarrow 0} 2x - 2 + \Delta x \\ \boxed{2x-2} \end{aligned}$$

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Chapter 2 Review

Limit: $f(c)$ doesn't matter
"Road" to $f(c)$

Notation:

$$\begin{aligned} \lim_{x \rightarrow c^-} &\rightarrow \text{left-side limit} \\ \lim_{x \rightarrow c^+} &\rightarrow \text{right-side limit} \\ \lim_{x \rightarrow c} &\rightarrow \lim_{x \rightarrow c^-} = \lim_{x \rightarrow c^+} \end{aligned}$$

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Finding Limits:

1. Direct Substitution
2. Factor and reduce
3. Rationalize Numerator - radicals
4. Piecewise functions
5. Infinite Limits $\rightarrow$ show points

* Study/Review Limit Properties

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

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Continuity:

3 conditions that must be met:

1. $f(c)$ is defined
2. $\lim_{x \rightarrow c} f(x)$ exists
3. $\lim_{x \rightarrow c} f(x) = f(c)$

Types:

1. Removable - hole - can patch
 $\rightarrow$ can factor and reduce
2. Nonremovable - jumps/asymptotes
 $\rightarrow$ cannot reduce out

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Vertical Asymptotes:

$$h(x) = \frac{f(x)}{g(x)} \quad \begin{matrix} f(x) \neq 0 \\ g(x) = 0 \end{matrix}$$

Intermediate Value Theorem: Continuous functions

$$f(a) < f(c) < f(b)$$

where $a < c < b$

* Make a statement

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Find the constants a and b such that the function is continuous on the entire real number line.

$$f(x) = \begin{cases} -10, & x \leq -2 \\ ax + b, & -2 < x < 3 \\ 5, & x \geq 3 \end{cases}$$

$$\begin{aligned} \lim_{x \rightarrow -2^-} &= \lim_{x \rightarrow -2^+} \\ -10 &= -2a + b \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 3^-} &= \lim_{x \rightarrow 3^+} \\ 3a + b &= 5 \end{aligned}$$

$$\begin{aligned} -2a + b &= -10 \\ 3a + b &= 5 \end{aligned}$$

$$\begin{aligned} a &= 3 \\ b &= -4 \end{aligned}$$

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