Synthetic Division

Synthetic division is another way to divide a polynomial by the binomial x - c, where c is a constant.

Step 1: Set up the synthetic division.

An easy way to do this is to first set it up as if you are doing long division and then set up your synthetic division.

The divisor (what you are dividing by) goes on the outside of the box. The dividend (what you are dividing into) goes on the inside of the box.

When you write out the dividend make sure that you write it in descending powers and you insert 0's for any missing terms. For example, if you had the problem $\left(x^4 - 3x + 5\right) \div (x - 4)$, the polynomial $x^4 - 3x + 5$, starts out with degree 4, then the next highest degree is 1. It is missing degrees 3 and 2. So if we were to put it inside a division box we would write it like this:

$$x - 4\overline{\big)x^4 + 0x^3 + 0x^2 - 3x + 5}.$$

This will allow you to line up like terms when you go through the problem.

When you set this up using synthetic division write c for the divisor x - c. Then write the coefficients of the dividend to the right, across the top. Include any 0's that were inserted in for missing terms.

$$\begin{array}{r|rrrrr} \text{c of divisor x - c} & \multicolumn{5}{l}{\text{coefficients of dividend}} \\ 4 & 1 & 0 & 0 & -3 & 5 \\ \hline & & & & & \end{array}$$

Step 2: Bring down the leading coefficient to the bottom row.

$$\begin{array}{r|rrrrr} 4 & 1 & 0 & 0 & -3 & 5 \\ & \downarrow & & & & \\ \hline & 1 & & & & \end{array}$$

Step 3: Multiply c by the value just written on the bottom row.

Place this value right beneath the next coefficient in the dividend:

$$\begin{array}{r|rrrrr} 4 & 1 & 0 & 0 & -3 & 5 \\ & & 4 & & & \\ \hline & 1 & & & & \end{array}$$

Step 4: Add the column created in step 3.

Write the sum in the bottom row:

$$\begin{array}{r|rrrrr} 4 & 1 & 0 & 0 & -3 & 5 \\ & & 4 & & & \\ \hline & 1 & 4 & & & \end{array}$$

Step 5: Repeat until done.

$$\begin{array}{r|rrrrr} 4 & 1 & 0 & 0 & -3 & 5 \\ & & 4 & 16 & 64 & 244 \\ \hline & 1 & 4 & 16 & 61 & 249 \end{array}$$

Step 6: Write out the answer.

The numbers in the last row make up your coefficients of the quotient as well as the remainder. The final value on the right is the remainder. Working right to left, the next number is your constant, the next is the coefficient for x, the next is the coefficient for x squared, etc...

The degree of the quotient is one less than the degree of the dividend. For example, if the degree of the dividend is 4, then the degree of the quotient is 3.

$$x^3 + 4x^2 + 16x + 61 + \frac{249}{x-4}$$

Example 1: Divide using synthetic division: $(2x^3 - 3x^2 + 4x - 1) \div (x + 1)$.

Step 1: Set up the synthetic division.

Long division would look like this:

$$x + 1 \overline{\smash{)}\,2x^3 - 3x^2 + 4x - 1}$$

Synthetic division would look like this:

c of divisor x - c — coefficients of dividend

$$\begin{array}{r|rrrr} -1 & 2 & -3 & 4 & -1 \\ & & & & \\ \hline & & & & \end{array}$$

Step 2: Bring down the leading coefficient to the bottom row.

$$\begin{array}{r|rrrr} -1 & 2 & -3 & 4 & -1 \\ & \downarrow & & & \\ \hline & 2 & & & \end{array}$$

*Bring down the 2

Step 3: Multiply *c* by the value just written on the bottom row.

$$\begin{array}{r|rrrr} -1 & 2 & -3 & 4 & -1 \\ & & -2 & & \\ \hline & 2 & & & \end{array}$$

*(-1)(2) = -2
*Place -2 in next column

Step 4: Add the column created in step 3.

$$\begin{array}{r|rrrr} -1 & 2 & -3 & 4 & -1 \\ & & -2 & & \\ \hline & 2 & -5 & & \end{array}$$

*-3 + (-2) = -5

Step 5: Repeat until done.

$$\begin{array}{r|rrrr} -1 & 2 & -3 & 4 & -1 \\ & & -2 & 5 & -9 \\ \hline & 2 & -5 & 9 & -10 \end{array}$$

Step 6: Write out the answer.

The numbers in the last row make up your coefficients of the quotient as well as the remainder. The final value on the right is the remainder. Working right to left, the next number is your constant, the next is the coefficient for *x*, the next is the coefficient for *x* squared, etc...

$$2x^2 - 5x + 9 - \frac{10}{x+1}$$

Example 2: Divide using synthetic division: $\dfrac{x^5 - 1}{x - 1}$

Step 1: Set up the synthetic division.

Long division would look like this:

$$x - 1\overline{)x^5 + 0x^4 + 0x^3 + 0x^2 + 0x - 1}$$

Synthetic division would look like this:

$$\begin{array}{r|rrrrrr} \text{c of divisor x - c} & \text{coefficients of dividend} & & & & & \\ 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ & & & & & & \\ \hline & & & & & & \end{array}$$

Step 2: **Bring down the leading coefficient** **to the bottom row.**

$$\begin{array}{r|cccccc} 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ & & & & & & \\ \hline & 1 & & & & & \end{array}$$

*Bring down the 1

Step 3: **Multiply *c* by the value just written** **on the bottom row.**

$$\begin{array}{r|cccccc} 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ & & 1 & & & & \\ \hline & 1 & & & & & \end{array}$$

*(1)(1) =1
*Place 1 in next column

Step 4: **Add the column** **created in step 3.**

$$\begin{array}{r|cccccc} 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ & & 1 & & & & \\ \hline & 1 & 1 & & & & \end{array}$$

*0 + 1 = 1

Step 5: **Repeat until done.**

$$\begin{array}{r|cccccc} 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ & & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 0 \end{array}$$

Step 6: **Write out the answer.**

The numbers in the last row make up your coefficients of the quotient as well as the remainder. The final value on the right is the remainder. Working right to left, the next number is your constant, the next is the coefficient for *x*, the next is the coefficient for *x* squared, etc...

$$x^4 + x^3 + x^2 + x + 1$$