Name: KEY Date: ____________________

Precalculus

Cum ulative Review #5 Due: ______________________

Directions: Show all work for full credit. Correct answers without supporting work will receive 1 credit.

**1.** Find $Cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$. domain $= 0° \le x \le 180°$

$\cos\theta = -\frac{\sqrt{2}}{2}$

$\theta = 45°$ III, II 225° 135°

$\theta = 135°$

**2.** Determine the equations of the vertical and horizontal asymptotes, if any, of $f(x) = \frac{3x}{x^2-1} = \frac{3x}{(x+1)(x-1)}$

$(x+1)(x-1)=0$

$x+1=0 \quad x-1=0$

$x=-1 \quad x=1$

$\dfrac{\frac{3x}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}} = \dfrac{\frac{3}{x}}{1-0} = 0$

$y=0$ ← reject

**3.** Write an equation of a sine function with amplitude 2, period 180°, and phase shift 45°. $y = A\sin(K\theta - c)$

Amp $= |A| = 2$, $A = \pm 2$

Per $= \frac{2\pi}{K} = \frac{360°}{K}$, $180° = \frac{360°}{K}$, $K = \frac{360}{180} = 2$

PS. $= \frac{-c}{K}$, $45° = -\frac{c}{2}$, $c = -90°$

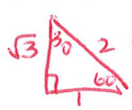
$y = A\sin(K\theta + c)$

$y = \pm 2\sin(2\theta - 90°)$

**4.** Find sin(Arctan $\sqrt{3}$) $-90 \le x \le 90$

.

$\tan\theta = \sqrt{3}$




$\theta = 60°$

$\sin 60° = \frac{\sqrt{3}}{2}$

**5.** State the amplitude, period, and phase shift for the function y = 8 cos(θ - 30°) .

$y = 8\cos(1\theta - 30°)$

$y = A\cos(K\theta - c)$

Amp $= |A| = |8| = 8$

Per $= \frac{2\pi}{K} = \frac{2\pi}{1} = 2\pi = 360°$

PS $= \frac{-c}{K} = \frac{+30°}{1}$ 30° Right or + 30°

**6.** Find the exact value of sec $\frac{\pi}{12}$ .

$\sec\frac{\pi}{12} = \sec 15° = \frac{1}{\cos 15°}$

OR $2\sqrt{2-\sqrt{3}}$ (when half ∡ used)

$\cos 15° = \cos(45-30) = \cos 45\cos 30 + \sin 45 \sin 30$
$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$
$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$
$= \frac{\sqrt{6}+\sqrt{2}}{4}$

$\sec = \frac{4(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} = \frac{4\sqrt{6}-4\sqrt{2}}{6-2}$
$= \sqrt{6}-\sqrt{2}$

**7.** Using a half-angle identity, find cot 67.5° .

$\tan 67.5 = \tan\left(\frac{135}{2}\right) = \oplus\sqrt{\frac{1-\cos 135}{1+\cos 135}}$

$= +\sqrt{\frac{1-(-\frac{\sqrt{2}}{2})}{1+(-\frac{\sqrt{2}}{2})}}$

$= +\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}$

$\frac{(2+\sqrt{2})(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} = \frac{(2+\sqrt{2})^2}{4-2} = \frac{(2+\sqrt{2})^2}{2}$

$= \sqrt{\frac{(2+\sqrt{2})^2}{2}}$

$= \frac{2+\sqrt{2}}{\sqrt{2}}$

$= \frac{2\sqrt{2}+2}{2}$

$= \sqrt{2}+1$

$\cot 67.5 = \frac{1(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = \frac{\sqrt{2}-1}{2-1} = \frac{\sqrt{2}-1}{1}$

$\sqrt{2}-1$

**8.** Verify that csc x cos x tan x = 1 is an identity.

$\frac{1}{\sin x} \cdot \frac{\cos x}{1} \cdot \frac{\sin x}{\cos x} = 1$

$1 = 1$ ✓

$\therefore \csc x \cos x \tan x = 1$

**9.** Find a numerical value of one trigonometric function of x if $\frac{\tan x}{\sec x} = \frac{\sqrt{2}}{5}$.

$\frac{\tan x}{\sec x} = \frac{\sqrt{2}}{5}$

$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = \frac{\sqrt{2}}{5}$

$\sin x = \frac{\sqrt{2}}{5}$

**10.** Solve $2\cos^2 x + 7\cos x - 4 = 0$ for $0 \le x \le 2\pi$.

$2x^2 + 7x - 4$

$(2x - 1)(x + 4)$

$(2\cos x - 1)(\cos x + 4) = 0$

$2\cos x - 1 = 0 \qquad \cos x + 4 = 0$

$\cos x = \frac{1}{2} \qquad \cancel{\cos x = -4}$

$\cos x = \frac{1}{2}$

$x = 60°$ in I, IV

$x = 60°, 300°$

**11.** If x and y are acute angles such that cos x = $\frac{1}{6}$ and cos y = $\frac{2}{3}$, find sin(x + y).

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$= \sin x \cdot \frac{2}{3} + \frac{1}{6} \cdot \sin y$

$= \frac{\sqrt{35}}{6} \cdot \frac{2}{3} + \frac{1}{6} \cdot \frac{\sqrt{5}}{3}$

$= \frac{2\sqrt{35}}{18} + \frac{\sqrt{5}}{18}$

$= \frac{2\sqrt{35} + \sqrt{5}}{18}$

$\cos x = \frac{1}{6}\frac{x}{r}$

$1^2 + y^2 = 6^2$

$y^2 = 35$

$y = \sqrt{35}$

$\sin x = \frac{\sqrt{35}}{6}$

$\cos y = \frac{2}{3}\frac{x}{r}$

$2^2 + y^2 = 3^2$

$y^2 = 5$

$y = \sqrt{5}$

$\sin y = \frac{\sqrt{5}}{3}$

**12.** Find the normal form of the equation -2x + 7y = 5 .

$-2x + 7y - 5 = 0$

$+\sqrt{(-2)^2 + 7^2}$

$+\sqrt{4 + 49}$

$+\sqrt{53}$

$\frac{-2x}{\sqrt{53}} + \frac{7y}{\sqrt{53}} - \frac{5}{\sqrt{53}} = 0$

$\frac{-2\sqrt{53}}{53}x + \frac{7\sqrt{53}}{53} - \frac{5\sqrt{53}}{53} = 0$

**13.** Find cos 2A if sin A = $\frac{\sqrt{3}}{6}$.

$\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\left(\frac{\sqrt{3}}{6}\right)^2$

$= 1 - 2\left(\frac{3}{36}\right)$

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$