

Name: KEY

Date: \_\_\_\_\_

Precalculus

Cumulative Review #2

Due: \_\_\_\_\_

Directions: Show all work for full credit. Correct answers without supporting work will receive 1 credit.

1. Use symmetry tests to determine whether the graph of  $y^3 - x^2 = 2$  is symmetric with respect to the x-axis, y-axis,  $y = x$ , and/or  $y = -x$ . If no symmetry exists, state "none".

x-axis:  $f(a,b) = f(a,-b)$

$f(a,b): b^3 - a^2 = 2$  so  $2 = b^3 - a^2$   
 $f(a,-b): (-b)^3 - a^2 = 2$   
 $-b^3 - a^2 = 2$  so  $2 = -b^3 - a^2$  } No symmetry

y-axis:  $f(a,b) = f(-a,b)$

$f(a,b): b^3 - a^2 = 2$  so  $2 = b^3 - a^2$   
 $f(-a,b): b^3 - (-a)^2 = 2$  so  $2 = b^3 - a^2$   
 $b^3 - a^2 = 2$  } Yes symmetry

2. Use a reflection matrix to find the coordinates of the vertices of a pentagon reflected over the y-axis if the coordinates of the vertices of the pentagon are (-5, -3), (-5, 4), (-3, 6), (-1, 3), and (-2, -2).

$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -5 & -5 & -3 & -1 & -2 \\ -3 & 4 & 6 & 3 & -2 \end{bmatrix} = \begin{bmatrix} -1(-5)+0(3) & -1(-5)+0(4) & -1(-3)+0(6) & -1(-1)+0(3) & -1(-2)+0(-2) \\ 0(-5)+1(3) & 0(-5)+1(4) & 0(-3)+1(6) & 0(-1)+1(3) & 0(-2)+1(-2) \end{bmatrix}$

↑ reflection matrix

$= \begin{bmatrix} 5 & 5 & 3 & 1 & 2 \\ -3 & 4 & 6 & 3 & -2 \end{bmatrix}$  or  $\begin{matrix} (5, -3) \\ (-5, 3) \\ (5, 4) \\ (3, 6) \\ (1, 3) \\ (2, -2) \end{matrix}$

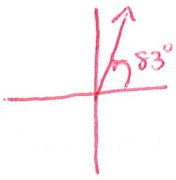
3. Find the value of the determinant for the matrix  $\begin{bmatrix} 4 & -3 & 2 \\ 8 & -2 & 0 \\ 9 & 6 & -3 \end{bmatrix}$ .

$4 \begin{vmatrix} -2 & 0 \\ 6 & -3 \end{vmatrix} + 3 \begin{vmatrix} 8 & 0 \\ 9 & -3 \end{vmatrix} + 2 \begin{vmatrix} 8 & -2 \\ 9 & 6 \end{vmatrix}$

$4(6) + 3(-24) + 2(66)$

$24 - 72 + 132 = 84$

4. Identify all angles that are coterminal with an  $83^\circ$  angle.



$$\left. \begin{array}{l} 83^\circ + 360n \\ 83^\circ - 360n \end{array} \right\} n \in \mathbb{Z} \text{ OR } \boxed{83^\circ + 360n, n \in \mathbb{Z}}$$

5. Suppose  $\theta$  is an angle in standard position whose terminal side lies in Quadrant

IV. If  $\sin \theta = -\frac{1}{6}$ , find the values of the remaining 5 trigonometric functions for  $\theta$ .

$$\begin{aligned} \sin \theta &= \frac{y}{r} = -\frac{1}{6} \\ x^2 + y^2 &= r^2 \\ x^2 + 1^2 &= 6^2 \\ x &= \sqrt{35} \end{aligned}$$

$$\sin \theta = -\frac{1}{6}$$

$$\csc \theta = \frac{r}{y} = -6$$

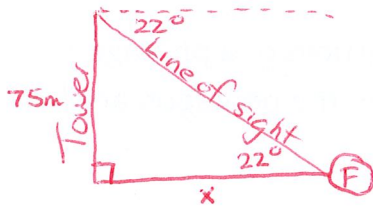
$$\cos \theta = \frac{x}{r} = \frac{\sqrt{35}}{6}$$

$$\sec \theta = \frac{r}{x} = \frac{6}{\sqrt{35}} = \frac{6\sqrt{35}}{35}$$

$$\tan \theta = \frac{y}{x} = \frac{-1}{\sqrt{35}} = -\frac{\sqrt{35}}{35}$$

$$\cot \theta = \frac{x}{y} = \frac{\sqrt{35}}{-1} = -\sqrt{35}$$

6. A fire is sighted from a fire tower in Wayne National Forest in Ohio. The ranger found that the angle of depression to the fire is  $22^\circ$ . If the tower is 75 ft tall, how far from the base of the tower is the fire?



$$\tan 22^\circ = \frac{75}{x}$$

$$x = \frac{75}{\tan 22^\circ}$$

$$x = 185.63 \approx 185.6 \text{ feet}$$

7. Using the inverse function test, determine whether the functions  $f(x) = \frac{3x}{x-1}$  and

$g(x) = \frac{x+1}{3x}$  are inverses of one another.

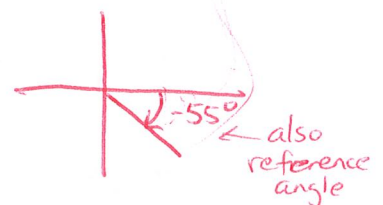
$$(f \circ g)(x) = \frac{3\left(\frac{x+1}{3x}\right)}{\frac{x+1}{3x} - 1} = \frac{\frac{x+1}{x}}{\frac{x+1-3x}{3x}} = \frac{\frac{x+1}{x}}{\frac{-2x+1}{3x}} = \frac{x+1}{x} \cdot \frac{3x}{-2x+1} = \frac{x+1}{-2x+1} \neq x$$

NOT inverses

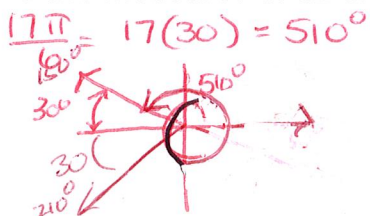
8. Find the reference angle for  $-775^\circ$

$$-775 + 360 = -415 + 360 = -55^\circ$$

$$\alpha = 55^\circ$$



9. Find the measures of all angles whose cosine value is the same as the  $\cos \frac{17\pi}{6}$ .



$$510^\circ - 360^\circ = 150^\circ$$

$$\text{So } 150^\circ + 360k \text{ and } 210^\circ + 360k, k \in \mathbb{Z}$$

OR

10. Pristine Pipes Inc. produces plastic pipe for use in newly-built homes. Two of the basic types of pipe have different diameters, wall thickness, and strengths. The strength of a pipe is increased by mixing a special additive into the plastic before it is molded. The table below shows the resources needed to produce 100 feet of each type of pipe and the amount of resources available each week.

Department	Pipe A	Pipe B	Resource Available
Extrusion	4 hrs	6 hrs	48 hrs
Packaging	2 hrs	2 hrs	18 hrs
Strengthening Additive	2 lbs	1 lb	16 lbs

$$\begin{aligned}
 4x + 6y &\leq 48 & 2x + 2y &\leq 18 \\
 6y &\leq -4x + 48 & 2y &\leq -2x + 18 \\
 y &\leq -\frac{4}{6}x + \frac{48}{6} & y &\leq -x + 9 \\
 y &\leq -\frac{2}{3}x + 8 & & \\
 2x + y &\leq 16 & & \\
 y &\leq -2x + 16 & &
 \end{aligned}$$

If the profit on 100 feet of type A pipe is \$34 and of type B pipe is \$40, how much of each should be produced to maximize profit?

$$\begin{aligned}
 P(x,y) &= 34x + 40y \\
 x &= \text{no. of 100' lengths of type A} \\
 y &= \text{no. of 100' lengths of type B}
 \end{aligned}$$

$$\begin{aligned}
 x \geq 0, y \geq 0 \\
 4x + 6y &\leq 48 \quad (y \leq -\frac{2}{3}x + 8) \\
 2x + 2y &\leq 18 \quad (y \leq -x + 9) \\
 2x + y &\leq 16 \quad (y \leq -2x + 16)
 \end{aligned}$$

Graph these

Vertices of polygon  
 $(0,0)$   $(8,0)$   $(7,2)$   
 $(0,8)$   $(3,6)$

$$\begin{aligned}
 \text{Test } (8,0): & 34(8) + 40(0) = 272 \\
 \text{Test } (7,2): & 34(7) + 40(2) = 318 \\
 \text{Test } (0,8): & 34(0) + 40(8) = 320 \\
 \text{Test } (3,6): & 34(3) + 40(6) = \mathbf{342 \text{ max}}
 \end{aligned}$$

$(3,6)$   
 300 ft of Type A  
 600 ft of Type B

11. If  $y$  varies inversely as the cube of  $x$  and directly as the square of  $z$  and  $y = -6$  when  $x = 3$  and  $z = 9$ , find  $y$  when  $x = 6$  and  $z = -4$ .

$$\begin{aligned}
 y &= \frac{kz^2}{x^3} \\
 -6 &= \frac{k(9^2)}{3^3} & -6 &= \frac{81k}{27} & y &= \frac{-2z^2}{x^3} \\
 -6 &= \frac{9k}{3} & -6 &= 3k & y &= \frac{-2(-4)^2}{6^3} = \frac{-2(16)}{216} = \frac{-32}{216} = \mathbf{\frac{-4}{27}} \\
 k &= -2
 \end{aligned}$$

12. The intensity of light  $I$ , measured in lux, is inversely proportional to the square of the distance  $d$  between the light source and the object illuminated.

a. Write an equation that represents this situation.

$$I = \frac{k}{d^2}$$

b. Using a light meter, a lighting director measures the intensity of the light from a bulb hanging 6 ft overhead a circular table at 16 lux. If the table has a 5 ft diameter, what illumination reading will the director find at the edge of the table where the actors will sit? Round to the nearest tenth.

$$\begin{aligned}
 16 &= \frac{k}{6^2} \\
 16 &= \frac{k}{36} \\
 k &= 16(36) = 576
 \end{aligned}$$

$$\begin{aligned}
 I &= \frac{576}{(\sqrt{42.25})^2} \\
 I &= \frac{576}{42.25} \\
 I &= \mathbf{13.6 \text{ Lux}}
 \end{aligned}$$

