Name: KEY Date: ____________

Precalculus

Cum ulative Review #4 Due: ____________

Directions: Show all work for full credit. Correct answers without supporting work will receive 1 credit.

1. State the amplitude, period, phase shift and vertical shift for $y = -3\cos(2\theta + \pi) + 5$.

$y = A\cos(K\theta - c) + h$

$A = -3,\ K = 2,\ c = -\pi,\ h = 5$

Amplitude $= |A| = |-3| = 3$
Period $= \frac{2\pi}{K} = \frac{2\pi}{2} = \pi$
Phase shift $= \frac{c}{K} = \frac{-\pi}{2}$
Vertical shift $= 5$

2. Decompose $\frac{2m+16}{m^2-16}$ into partial fractions.

$\frac{2m+16}{(m+4)(m-4)} = \frac{A}{m+4} + \frac{B}{m-4}$

$\frac{2m+16}{m^2-16} = \frac{-1}{m+4} + \frac{3}{m-4}$

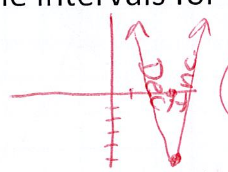
$2m + 16 = A(m-4) + B(m+4)$

Let $m = 4$:
$2(4) + 16 = A(0) + B(8)$
$24 = 8B$ so $B = 3$

Let $m = -4$:
$2(-4) + 16 = A(-4-4) + B(-4+4)$
$-8 + 16 = A(-8) + B(0)$
$8 = -8A$
$A = -1$

3. Determine the intervals for which the graph of $f(x) = 2|x-3| - 5$ is increasing and the intervals for which the graph is decreasing.

vertex @ $(3,-5)$



Decreasing $(-\infty, 3)$
Increasing $(3, +\infty)$

4. If a central angle of a circle with radius 18 cm measures $\frac{\pi}{3}$, find the length (in terms of π) of its intercepted arc.

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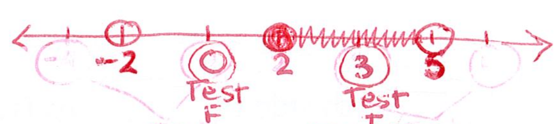
$S = r\theta$
$S = 18\text{cm}\left(\frac{\pi}{3}\right) = 6\pi\text{cm}$

5. Solve $\frac{x^2-4}{x^2-3x-10} < 0$.

$\frac{(x+2)(x-2)}{x^2-4}$ $x+2=0, x=-2$ $x-2=0$ $x=2$

$(x-5)(x+2)$

$x-5=0$ $x+2=0$
$x=5$ $x=-2$ ← excluded values)



Test 0: $\frac{0^2-4}{0^2-3(0)-10} < 0?$
$\frac{-4}{-10} = \frac{2}{5}$ F

Test 3: $\frac{3^2-4}{3^2-3(3)-10} = \frac{9-4}{9-9-10} = \frac{5}{-10} = \frac{-1}{2}$ T

$2 \le x < 5$

6. Write the equation of a sine function with amplitude 5, period 3π, phase shift –π, and vertical shift -8.

$y = A\sin(K\theta - c) + h$

$y = \pm 5\sin\left(\frac{2}{3}\theta + \frac{2}{3}\pi\right) - 8$

Per $= \frac{2\pi}{K}$
$3\pi = \frac{2\pi}{K}$
$K = \frac{2\pi}{3\pi} = \frac{2}{3}$

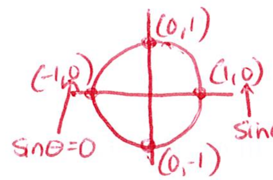
P.S. $= -\frac{c}{K}$
$-\pi = \frac{-c}{\frac{2}{3}}$
$-c = -\frac{2}{3}\pi$
$c = \frac{2}{3}\pi$

7. What are the values of θ for which csc θ is undefined?

$\csc\theta = \frac{1}{\sin\theta}$

so if $\sin\theta = 0$

then $\csc\theta$ = undefined



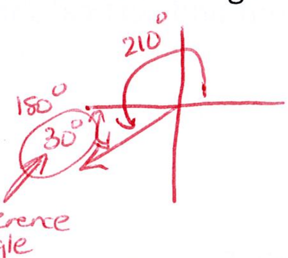
so since $\sin\theta = 0$ @ $0°$ & $180°$

$\csc\theta$ is undefined @ $0°$, $180°$

$0° + \pi n = \pi n, n \in \mathbb{Z}$

8. Find the measure of the reference angle for an angle of $\frac{7\pi}{6}$

$\frac{7\pi}{6} = 210°$



$\alpha = 30°$

9. Graph y = cos x for -11π < x < -9π.

$-11\pi < x < -9\pi$

$+2\pi \qquad +2\pi$ ← 1 rotation

$-9\pi < x < -7\pi$

$+2\pi \qquad +2\pi$ ←

$-7\pi < x < -5\pi$

$+2\pi \qquad +2\pi$ ←

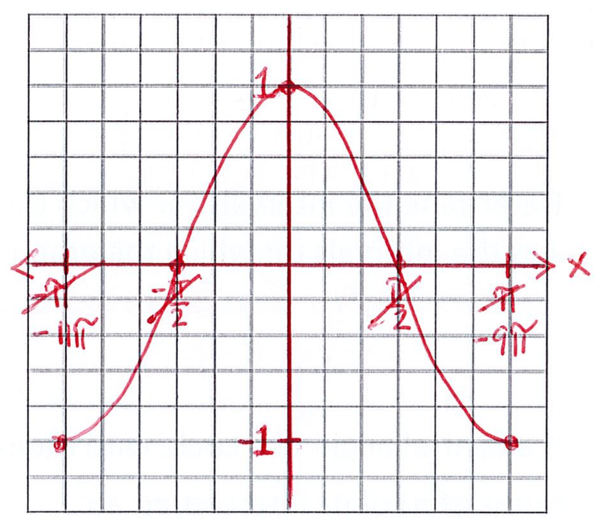
$-5\pi < x < -3\pi$

$+2\pi \qquad +2\pi$ ←

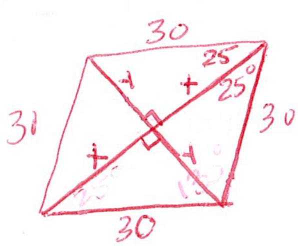
$-3\pi < x < -1\pi$

$+2\pi \qquad +2\pi$ ←

$-\pi < x < \pi$ ← graph this



10. Each side of a rhombus is 30 units long. One diagonal makes a 25° angle with a side. What is the length of each diagonal to the nearest tenth of a unit?



$\sin 25° = \frac{y}{30}$

$y = 30\sin 25°$

$y = 12.6785$

$2(12.6785)$

$= 25.4$

$\cos 25 = \frac{x}{30}$

$x = 30\cos 25$

$x = 27.1892$

$2(27.1892)$

$= 54.4$

11. In Daytona Beach FL, the first high tide was 3.99 feet at 12:03 am. The first low tide of 0.55 foot occurred at 6:24 am. The second high tidel occuured at 12:19 pm.

a. Find the amplitude of a sinusoidal function that models the tides.

$A = \frac{3.99 - 0.55}{2} = 1.72 \text{ ft}$

b. Find the vertical shift of a sinusoidal function that models the tides.

$A = \frac{3.99 + 0.55}{2} = 2.27 \text{ ft}$

c. What is the period of a sinusoidal function that models the tides?

Period = High tide #2 − Hight tide #1

= 12:19pm − 12:03am = 12.2667 ≈ 12.3 hrs.

d. Write a sinusoidal function to model the tides, using t to represent the number of hours in decimals since midnight.

$h(t) = 1.72 \cos\left(\frac{2\pi}{12.3} t\right) + 2.27$

$Per = \frac{2\pi}{K}$

$K = \frac{2\pi}{Per}$

$K = \frac{2\pi}{12.3}$

e. According to your model, determine the height of the water at noon.

noon = 12 hrs. since midnight so $t = 12$

$h(12) = 1.72 \cos\left(\frac{2\pi}{12.3} \cdot 12\right) + 2.27$

$h(12) \approx 3.98 \text{ ft}$

12. If r varies directly as t and $t = 6$ when $r = 0.5$, find r when $t = 10$.

$r = Kt$

$0.5 = K(6)$

$0.5 = 6K$

$K = \frac{0.5}{6}$

$K = \frac{1}{12}$

$r = \frac{1}{12} t$

$r = \frac{1}{12}(10)$

$r = \frac{10}{12} = \frac{5}{6}$

13. What is the inverse of $\begin{bmatrix} \frac{3}{4} & -\frac{1}{8} \\ 5 & \frac{1}{2} \end{bmatrix}$?

$\frac{1}{\begin{vmatrix} \frac{3}{4} & -\frac{1}{8} \\ 5 & \frac{1}{2} \end{vmatrix}} \begin{bmatrix} \frac{1}{2} & \frac{1}{8} \\ -5 & \frac{3}{4} \end{bmatrix} = \frac{1}{1} \begin{bmatrix} \frac{1}{2} & \frac{1}{8} \\ -5 & \frac{3}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{8} \\ -5 & \frac{3}{4} \end{bmatrix}$

$\frac{3}{4}\left(\frac{1}{2}\right) - 5\left(-\frac{1}{8}\right)$

$\frac{3}{8} + \frac{5}{8} = \frac{8}{8} = 1$